Reply by Author to H. Guthart

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IN his comments Guthart has made a very neat derivation of the output spectrum of a sensor responsive to a plane of infinite extent and of infinitesimal thickness normal to the convection velocity of a homogenous turbulent flow. The author is familiar with this derivation through correspondence[1] with Guthart and pointed out[2] that his expression for $E_p(n)$ [see Eq. (8)] should reduce to a hot wire or "point" spectrum when $A(r)$ is a delta function. The author certainly agrees that a "point" resolution device, e.g., a hot wire probe, would respond better at high wavenumbers than the radiometer, which has comparatively poorer spatial resolution and operates as a low-pass spatial filter. This in fact was demonstrated experimentally in the author's paper[3] with regard to changes in the distance along the jet axis which the radiometer viewed (called the "aperture width," see Fig. 1 of Ref. 3). Thus the radiometer output spectrum was shown to have been processed by the low-pass filter function shown in Fig. 11 of Ref. 3.

The problem at hand is the validity of employing the hot wire spectrum as the radiometric spectrum of a thin turbulent jet cross section. Prior to studying the hot-jet radiometric data, an exact solution of the radiometric spectrum seemed necessary. However, the radiometric spectra obtained displayed a strong dependence on the radiometer aperture width (size and shape) used and in the limit of the "thin" cross section (infinitesimal aperture width w), these spectra closely resembled the hot wire temperature spectra of Corrsin and Uberoi.[4] The reduced spectral data (energy density vs wavenumber k) show the high-wavenumber spectra behavior varying from k^{-5} to $k^{-2.3}$ as the radiometric aperture width is decreased. Extrapolation of this data to an infinitesimal radiometric aperture width indicated corresponding high-wavenumber slopes behaving as k^{-2}.

While discussing these high-wavenumber slopes, it is worth pointing out that the radiometric aperture shapes (as distinguished from the width) are important. A rectangular radiometric aperture shape (such as used in Ref. 3) attenuates high wavenumbers with a k^{-1} filtering process. But any realistic aperture is rounded off (e.g., a Gaussian pulse). It "contains" relatively fewer high-frequency components than the rectangular pulse and suppresses high wavenumbers more severely (e.g., a Gaussian pulse Fourier transforms into a Gaussian pulse). Thus the very steep high wavenumber shapes k^{-5} previously referred to are due to the radiometric aperture shape and do not indicate that the hot-jet radiation from a thin slab normal to the jet axis generates a radiometric spectrum with shape k^{-4} rather than k^{-2}.

In any event, the experimental turbulent hot-jet spectra do not seem to behave as k^{-4} as suggested by Guthart. The author feels that this disparity between Guthart's results using infinite limits and experiment arises from the limited extent of the jet. The radiating jet region is not infinite in extent with respect to the jet local transverse correlation length. Generally we expect the instrument with the poorer spatial resolution to suppress the high-wavenumber radiometric spectra more than the point resolution instrument, except in the case when both instruments are viewing areas over which the spatial information is strongly correlated. In the case of radiometrically viewing a very thin slab perpendicular to the hot-jet axis, the ratio of the effective radiating cross section of the hot jet [based on Guthart's $A(r)$] to the transverse temperature correlation length becomes the parameter that indicates whether the radiometric spectrum behaves as k^{-4} when the parameter is infinite (uncorrelated flow), or as $k^{-5/3}$ when the parameter is zero (correlated flow.)

This can be shown by using Guthart's expression for $E_p(n)$, letting $A(x)$ be a delta function, normalizing lengths by a transverse temperature correlation length Λ_y, equal to longitudinal temperature correlation length Λ_{xi} and representing the upper limit in r by R

$$E_p(n) \propto \int_0^{R'} \frac{A(r')K_1\{r'[(1 + S^2)]^{1/2}\}}{(1 + S^2)^{1/2}} r'^2 \, dr'$$

where

$(\ )' = (\ )/\Lambda$
S = reduced frequency $= 2\pi n\Lambda/\tilde{U}R\Lambda = k\Lambda$
$A(x) = \delta(x)$
R = maximum jet radius of interest
$\Lambda_y = \Lambda_x = \Lambda$
$A(r') = 1$

We let the reduced frequency become very large so $S \gg 1$. Then, for $R' \gg 1$ or "uncorrelated" flow $E_p(n) \propto k^{-4}$; for $R' \ll 1$ or "correlated" flow, $E_p(n) \propto k^{-2}$.

Since the hot wire spectrum lies closer to $k^{-5/3}$ than to k^{-2}, it appears that the $k^{-2} - k^{-2.2}$ behavior observed by the author[3] for thin ($w/\Lambda \ll 1$) aperture widths very likely indicates an intermediate case where the transverse temperature correlation length Λ is on the order of the flow viewed R.

References

[1] Guthart, H., private communication (November 18, 1966).
[2] Draper, J. S., private communication (January 10, 1967).
[3] Draper, J. S., "Infrared radiometry of turbulent flows," AIAA J. 4, 1957–1603 (1966).
[4] Corrsin, S. and Uberoi, M. S., "Spectra and diffusion in a round turbulent jet," NACA Rept. 1040 (1951).

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Comment on "Ballistic Coefficients for Power Law Bodies"

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BALLISTIC coefficients of slender, axisymmetric, power law bodies of revolution were reported by Berman[1] for constant values of length, base diameter, and specific weight. The ratio of ballistic coefficient of a power law body to that of a sharp cone was computed from an empirical variation of drag coefficient with power law exponent as determined from wind-tunnel data and limited inviscid theoretical calculations. A maximum ballistic coefficient 42% larger than that of a sharp cone was determined for a power law exponent of 0.62. It would be interesting to know whether the same optimum solution could have been determined analytically from Newtonian theory.

Newtonian impact theory and Newtonian theory with the Busemann centrifugal correction give infinite drag coefficients for slender power law bodies with exponents less than or equal to $\frac{1}{2}$. Taking this as a lower limit on the exponent n, the ratio of ballistic coefficients derived from New-

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tonian impact theory for slender bodies of revolution can be shown to be given by

$$\beta/\beta_{\text{cone}} = 3(2n - 1)/[n^3(2n + 1)] \tag{1}$$

The maximum value of this ratio is

$$(\beta/\beta_{\text{cone}})_{\max} = 6^4[(10)^{1/2} - 2]/\{2[(10)^{1/2} + 4][(10)^{1/2} + 1]^3\} \tag{2}$$

or about 1.458, at a power law exponent given by

$$n = [(10)^{1/2} + 1]/6 \simeq 0.694 \tag{3}$$

Thus the maximum empirical value of ballistic coefficient is overestimated by about 4% and the optimum exponent is overestimated by about 10% with Newtonian impact theory.

The ratio of ballistic coefficients obtained from Newton-Busemann centrifugal theory under the same assumptions is

$$\beta/\beta_{\text{cone}} = 6(2n - 1)/[n^2(3n - 1)(2n + 1)] \tag{4}$$

This ratio is approximately equal to 1.952 at the optimum power law exponent of about 0.637. As with the results published in Ref. 2, the optimized aerodynamic quantity obtained from higher-order hypersonic theory is fairly close to that calculated from Newtonian impact theory but differs markedly from that of Newtonian centrifugal theory.

Subsequent to this analysis, optimum slender bodies that satisfy this constraint for a Newtonian impact pressure distribution and constant skin friction coefficient have been derived by Miele and Huang.[3]

References

[1] Berman, R. J., "Ballistic coefficients for power law bodies," AIAA J. **5**, 166–167 (1967).

[2] Fink, M. R., "Hypersonic minimum-drag slender bodies of revolution," AIAA J. **4**, 1717–1724 (1966).

[3] Miele, A. and Huang, H.-Y., "Missle shapes of minimum ballistic factor," Rice Univ. Aero Astronautics Rept. 32 (April 1967; also J. Optimization Theory Appl. (to be published).

Announcement: Change in Style for References in AIAA Publications

The Committee of Engineering Society Editors, of the Engineers Joint Council, has recommended a standard style for references in engineering publications. In the interest of reducing the burden on authors and editors and minimizing confusion, the AIAA Publications Department has decided to follow the recommended style. Examples of the new style will be found below and on the inside back cover of all AIAA journals. The changes will be effective with manuscripts scheduled for the January 1968 issues and thereafter.

Example—Journals

Walker, R. E., Stone, A. R., and Shandor, M., "Secondary Gas Injection in a Conical Rocket Nozzle," *AIAA Journal,* Vol. 1, No. 2, Feb. 1963, pp. 334–338.

Examples—Books

Turner, M. J., Martin, H. C., and Leible, R. C., "Further Development and Applications of Stiffness Method," *Matrix Methods of Structural Analysis,* 1st ed., Vol. 1, Macmillan, New York, 1964, pp. 203–266.

Segrè, E., ed., *Experimental Nuclear Physics,* 1st ed., Vol. 1, Wiley, New York, 1953, pp. 6–10.

Example—Reports

Book, E. and Bratman, H., "Using Compilers to Build Compilers," SP-176, Aug. 1960, Systems Development Corp., Santa Monica, Calif.

Example—Transactions or Proceedings

Soo, S. L., "Boundary Layer Motion of a Gas-Solid Suspension," *Proceedings of the Symposium on Interaction between Fluids and Particles,* Institute of Chemical Engineers, Vol. 1, 1962, pp. 50–63.